[2 PTS] Write the vertex form of the equation of the quadratic function that has vertex $(-1, 2)$ and whose graph passes through the point $(-3, 5)$.

ANSWER: $f(x) = \frac{3}{4}(x+1)^2 + 2$

$f(x) = a(x - -1)^2 + 2 = a(x+1)^2 + 2$

$f(-3) = a(-3+1)^2 + 2 = 4a + 2 = 5$

$a = \frac{3}{4}$

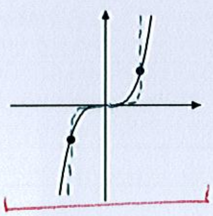
[3 PTS] Divide $f(x) = 3x + 2x^3 - 9 - 8x^2$ by $d(x) = x^2 + 1$.

Write your final answer in the form $q(x) + \dfrac{r(x)}{d(x)}$.

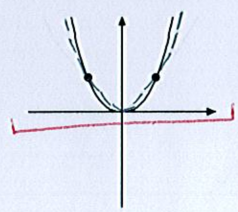
ANSWER: $2x - 8 + \dfrac{x-1}{x^2+1}$

$$
\begin{array}{r}
2x - 8 \\
x^2 + 1 \overline{)\, 2x^3 - 8x^2 + 3x - 9} \\
\underline{2x^3 + 2x } \\
-8x^2 + x - 9 \\
\underline{-8x^2 - 8} \\
x - 1
\end{array}
$$

[2 PTS] [a] The graph of $f(x) = x^3$ is shown below with the points $(1, 1)$ and $(-1, -1)$ highlighted. On the same grid, sketch the graph of $f(x) = x^5$.



[b] The graph of $f(x) = x^4$ is shown below with the points $(1, 1)$ and $(-1, 1)$ highlighted. On the same grid, sketch the graph of $f(x) = x^2$.



[2 PTS] Find the vertex and axis of symmetry of $f(x) = \frac{1}{4}x^2 - 2x - 12$.

ANSWER: vertex: $(4, -16)$

axis: $x = 4$

$x = -\dfrac{b}{2a} = -\dfrac{-2}{2(\frac{1}{4})} = 4$

$f(4) = \frac{1}{4}(4)^2 - 2(4) - 12 = -16$

ADDITIONAL QUESTIONS ON THE OTHER SIDE ➡

[4 PTS] $(x-5)$ and $(x+4)$ are factors of $f(x)=x^4-4x^3-15x^2+58x-40$. **ANSWER:** $5, -4, 1, 2$

Using that information and synthetic division, find all real zeros of f.

$$\begin{array}{r|rrrrr} 5 & 1 & -4 & -15 & 58 & -40 \\ & & 5 & 5 & -50 & 40 \\ \hline -4 & 1 & 1 & -10 & 8 & \boxed{0} \\ & & -4 & 12 & -8 & \\ \hline & 1 & -3 & 2 & \boxed{0} & \end{array}$$

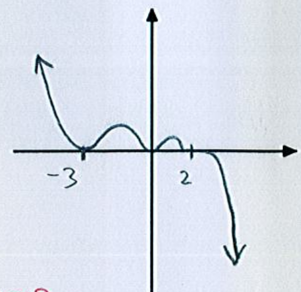
OR

$$\begin{array}{r|rrrrr} -4 & 1 & -4 & -15 & 58 & -40 \\ & & -4 & 32 & -68 & 40 \\ \hline 5 & 1 & -8 & 17 & -10 & \boxed{0} \\ & & 5 & -15 & 10 & \\ \hline & 1 & -3 & 2 & \boxed{0} & \end{array}$$

$x^2-3x+2=0$

$(x-1)(x-2)=0 \rightarrow x=1,2$

[3 PTS] Sketch the graph of the function $f(x)=x^2(x+3)^2(2-x)^5$ as shown in lecture. **ANSWER:**



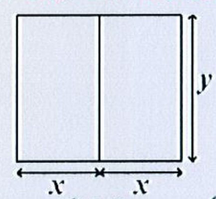
ZERO	MULT	
0	2	BOUNCE
-3	2	BOUNCE
2	5	CROSS

LEADING TERM $-x^9$

BEHAVIOR
AT $x=-3$ ①
$x=0$ ①
$x=2$ ①

(-1) ANY OTHER ZEROS
(-1) WRONG LONG RUN
(-1) NOT SMOOTH + CONTINUOUS

[4 PTS] A rancher has 100 feet of fencing to enclose two adjacent rectangular corrals (see figure on the right).



[a] Write the total (combined) enclosed area of the corrals as a function of x. (Your final answer must **NOT** involve y.) **ANSWER:** $\frac{200}{3}x-\frac{8}{3}x^2$

$A=2xy \qquad 4x+3y=100$

$y=\frac{100-4x}{3}$

$A=2x\left(\frac{100-4x}{3}\right)=\frac{200}{3}x-\frac{8}{3}x^2$

[b] Find the dimensions of **each** corral that will produce the maximum enclosed area. **ANSWER:** $x=\frac{25}{2}, y=\frac{50}{3}$

VERTEX $x=-\frac{b}{2a}=-\frac{\frac{200}{3}}{2(-\frac{8}{3})}=\frac{200}{3}\cdot\frac{3}{16}=\frac{25}{2}$

$y=\frac{100-4\left(\frac{25}{2}\right)}{3}=\frac{50}{3}$